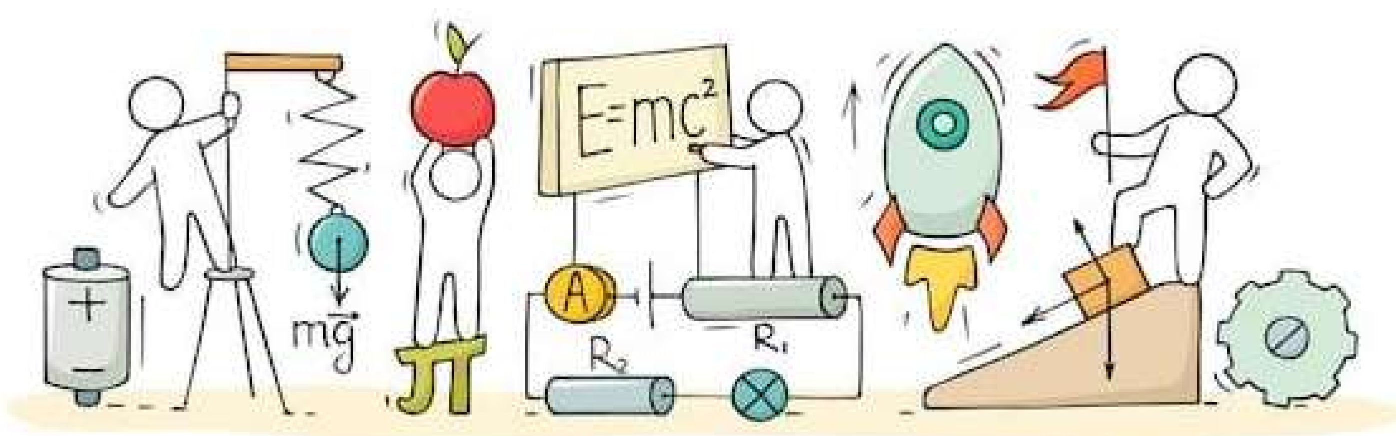
PHYSICS

Chapter 14: Oscillations



Oscillations

Top Formulae

Displacement in SHM	$y = a \sin(\omega t \pm \phi_0)$
Velocity in SHM	$v = \omega \sqrt{a^2 - y^2}$
Acceleration in SHM	$a = -\omega^2 y$ and $\omega = 2\pi\nu = 2\pi/T$
Potential energy in SHM	$U = \frac{1}{2} m \omega^2 a^2 = \frac{1}{2} k y^2$
Kinetic energy in SHM	$K = \frac{1}{2} m \omega^2 (a^2 - y^2) = \frac{1}{2} k (a^2 - y^2)$
Total energy	$E = \frac{1}{2} m \omega^2 a^2 = \frac{1}{2} k a^2$
Spring constant	$k = F/y$
Spring constant of parallel combination of springs	$k = k_1 + k_2$
Spring constant of series combination of springs	$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k}$
Time period	$T = 2\pi \sqrt{\frac{m}{K}}$
Equation of displacement in damped oscillation	If the damping force is given by $F_d = -b v$, where v is the velocity of the oscillator and b is its damping constant, then the displacement of the oscillator is given by $x(t) = A e^{-bt/2m} \cos(\omega' t + \Phi)$, where ω' is the angular frequency of the damped oscillator and is given by $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
Mechanical energy E of damped oscillator	$E(t) = \frac{1}{2} k A^2 e^{-bt/m}$

Top Concepts

- The motion which repeats itself is called periodic motion.
- The period T is the time required for one complete oscillation or cycle. It is related to the frequency by $\nu = 1/T$.
- The frequency ν of periodic or oscillatory motion is the number of oscillations per unit time.
- The force acting in simple harmonic motion is proportional to displacement and is always directed towards the centre of motion.
- In simple harmonic motion, the displacement x (t) of a particle from its equilibrium position is given by $x(t) = A\cos(\omega t + \Phi)$
- $(\omega t + \Phi)$ is the phase of the motion and Φ is the phase constant. The angular frequency ω is related to the period and frequency of the motion by

$$\omega = \frac{2\pi}{T} = 2\pi\upsilon$$

- Two perpendicular projections of uniform circular motion will give simple harmonic motion for projection along each direction with the centre of the circle as the mean position.
- The motion of a simple pendulum swinging through small angles is approximately simple harmonic. The period of oscillation is given by $T = 2\pi\sqrt{\frac{l}{g}}$
- The motion of a simple pendulum is simple harmonic for small angular displacement.
- A particle of mass *m* oscillating under the influence of a Hooke's law restoring force given by $F = -kx$ exhibits simple harmonic motion with

$$\omega = \sqrt{\frac{k}{m}} = \text{Angular frequency}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = \text{Period}$$

- The restoring force in case of a wooden cylinder floating on water is due to an increase in upthrust as it is pressed into the water.

- The restoring force in case of a liquid in a U-tube arises due to excess pressure in the liquid column when the liquid levels in the two arms are not equal.

- A simple pendulum undergoing SHM in the plane parallel to the length of the wire is due to the restoring force which arises due to increase in tension in the wire.

- The mechanical energy in a real oscillating system decreases during oscillations because external forces, such as drag, inhibit the oscillations and transfer mechanical energy to thermal energy. The real oscillator and its motion are then said to be damped.

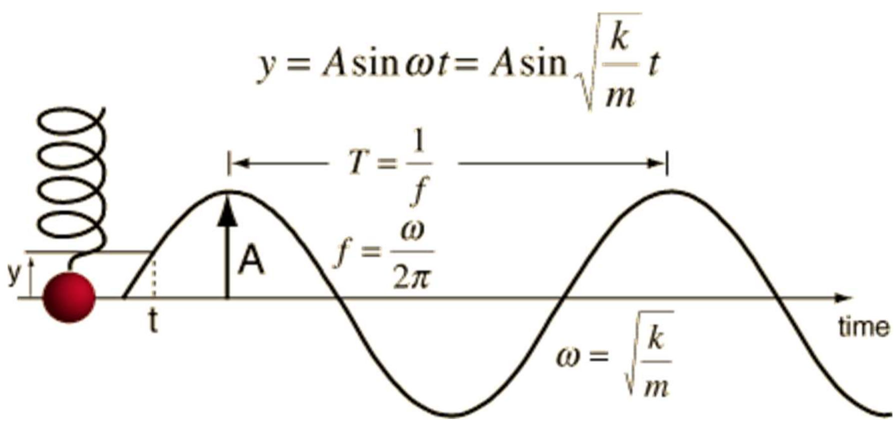
- If an external force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω_d, Then the system oscillates with angular frequency ω_d. The amplitude of oscillations is the greatest when

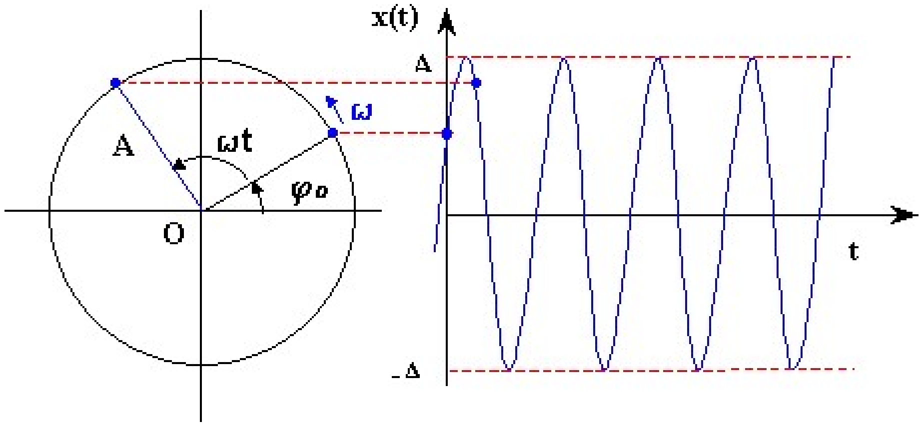
  $\omega_d = \omega$

  This condition is called resonance.

Diagrams

Simple harmonic motion




x(t)
A
ω
A
ωt
φ0
O
t
-A

CHAPTER - 14 : OSCILLATIONS

Oscillations

- Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.
- To and fro motion repeatedly about a fixed point in a definite interval of time.

SHM

Simple Harmonic Motion is the simplest form of oscillatory motion

Displacement in SHM

$x(t) = A\cos(\omega t+\phi)$

A = Amplitude i.e., maximum displacement of particles.

Phase

Time varying quantity $(\omega t+\phi)$

Phase constant or Phase angle (ϕ)
- It depends upon velocity (v) and displacement of particle at t = 0

Time period (T)

The smallest interval of time after which the motion is repeated

Frequency

The number of oscillations per second i.e., $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Velocity in SHM

$v = \frac{dx(t)}{d(t)} = -\omega A\sin(\omega t+\varphi)$

$v_{max} = \omega A$

Acceleration in SHM

$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t+\varphi)$

$a_{max} = \omega^2 A$

Energy in SHM

Kinetic Energy

K.E = $m\omega^2 A^2 \sin^2(\omega t+\varphi)$

Potential Energy

P.E = $\frac{1}{2} m\omega^2 A^2 \cos^2(\omega t+\varphi)$

Total energy, T.E. = K.E. + P.E.

$= \frac{1}{2} kA^2[\cos^2(\omega t+\varphi)+\sin^2(\omega t+\varphi)] = \frac{1}{2}KA^2$

Force law equation for SHM

$F = -k(x), k = m\omega^2$

$\therefore \omega = \sqrt{\frac{k}{m}}$

Simple pendulum

Time period $T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$

Oscillations due to a spring

Time period $T = 2\pi\sqrt{\frac{m}{k}}$

Here, $k = \frac{F}{x}$ = *spring constant*

Damped Oscillations

Oscillation of a body whose amplitude goes on decreasing with time.

Damping force $F_d = -bv$, b = damping constant

Displacement of damped oscillator

$x = x_m e^{-bt/2m}\sin(\omega' t+\varphi)$, here, $\omega' = \sqrt{\frac{m}{k} - \frac{b^2}{4m^2}}$

Amplitude decreases with time as

$x = x_m e^{-(b/2m)t}$

Forced Oscillations

Driving force, $F(t) = F_o\cos\omega_d t$

Displacement, $x(t) = A\cos(\omega_d t+\varphi)$

Amplitude, $A = \frac{F_o}{\left\{m^2(\omega^2-\omega_d^2)^2+\omega_d^2 b^2\right\}^{\frac{1}{2}}}$

$\tan\varphi = \frac{-v_o}{\omega_d x_o}$

Resonance

When frequency of external force is equal to natural frequency of oscillator.

Important Questions

Multiple Choice questions-

Question 1. If an simple pendulum oscillates with an amplitude of 50 mm and time period of 2s, then its maximum velocity is

(a) 0.10 m/s

(b) 0.16 m/s

(c) 0.25 m/s

(d) 0.5 m/s

Question 2. If the frequency of the particle executing S.H.M. is n, the frequency of its kinetic energy becoming maximum is

(a) n/2

(b) n

(c) 2n

(d) 4n

Question 3. Spring is pulled down by 2 cm. What is amplitude of motion?

(a) 0 cm

(b) 6 cm

(c) 2 cm

(d) cm

Question 4. The period of thin magnet is 4 sec. if it is divided into two equal halves then the time period of each part will be

(a) 4 sec

(b) 1 sec

(c) 2 sec

(d) 8 sec

Question 5. The acceleration of particle executing S.H.M. when it is at mean position is

(a) Infinite

(b) Varies

(c) Maximum

(d) Zero

Question 6. A spring of force constant k is cut into two pieces such that on piece is double the length of the other. Then the long piece will have a force constant of

(a) 2 k/3

(b) 3 k/2

(c) 3 k

(d) 6 k

Question 7. Particle moves from extreme position to mean position, its

(a) Kinetic energy increases, potential increases decreases

(b) Kinetic energy decreases, potential increases

(c) Both remains constant

(d) Potential energy becomes zero and kinetic energy remains constant

Question 8. Grap of potential energy vs. displacement of a S.H. Oscillator is

(a) parabolic

(b) hyperbolic

(c) elliptical

(d) linear

Question 9. The time-period of S.H.O. is 16 sec. Starting from mean position, its velocity is 0.4 m/s after 2 sec. Its amplitude is

(a) 0.36 m

(b) 0.72 m

(c) 1.44 m

(d) 2.88 m

Question 10. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will

(a) Remain unchanged

(b) Increase

(c) Decrease

(d) Become erratic

Assertion Reason Questions:

1. **Directions:**

   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false

Assertion: Sine and cosine functions are periodic functions.
Reason: Sinusoidal functions repeats it values after a definite interval of time.

2. **Directions:**

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false

Assertion: Simple harmonic motion is a uniform motion.
Reason: Simple harmonic motion is not the projection of uniform circular motion.

MCQ Answers-

1. Answer: (b) 0.16 m/s
2. Answer: (c) 2n
3. Answer: (c) 2 cm
4. Answer: (c) 2 sec
5. Answer: (d) Zero
6. Answer: (b) 3 k/2
7. Answer: (a) Kinetic energy increases, potential increases decreases
8. Answer: (a) parabolic
9. Answer: (c) 1.44 m
10. Answer: (b) Increase

Very Short Questions-

1. How is the time period effected, if the amplitude of a simple pendulum is in Creased?

2. Define force constant of a spring.

3. At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy?

4. How is the frequency of oscillation related with the frequency of change in the of K. E and PE of the body in S.H.M.?

5. What is the frequency of total energy of a particle in S.H.M.?

6. How is the length of seconds pendulum related with acceleration due gravity of any planet?
7. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), then time period of the pendulum increased or decrease.

8. How is the time period of the pendulum effected when pendulum is taken to hills Or in mines?

9. Define angular frequency. Give its S.I. unit.

10. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?

Very Short Answers-

1. **Ans.** No effect on time period when amplitude of pendulum is increased or decreased.
2. **Ans.** The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.
3. **Ans.** Not at the mid-point, between mean and extreme position. it will be at $x = a\sqrt{2}.$
4. **Ans.** P.E. or K.E. completes two vibrations in a time during which S.H.M completes one vibration or the frequency of P.E. or K.E. is double than that of S.H.M
5. **Ans.** The frequency of total energy of particle is S.H.M is zero because it retain constant.
6. **Ans.** Length of the seconds pendulum proportional to acceleration due to gravity)
7. **Ans.** Increased

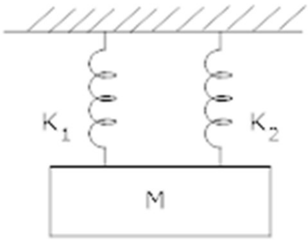
8. Ans. As $T \alpha \frac{1}{\sqrt{g}}$, T will increase.

9. Ans. It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π.
$\omega = 2\pi v$, S.I. unit is rads s^{-1}

10.Ans. No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

Short Questions-

1.A mass = m suspend separately from two springs of spring constant k_1 and k_2 gives time period t_1 and t_2 respectively. If the same mass is connected to both the springs as shown in figure. Calculate the time period 't' of the combined system?



2.Show that the total energy of a body executing SHN is independent of time?

3.A particles moves such that its acceleration 'a' is given by a = -b x where x = displacement from equilibrium position and b is a constant. Find the period of oscillation? 2

4.A particle is S.H.N. is described by the displacement function: →

$$x = A \text{ Cos } (wt + \Phi); w = \frac{2\pi}{T}$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm | s, What are its amplitude and phase angle?

5.Determine the time period of a simple pendulum of length = l when mass of bob = m Kg? 3

6. Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

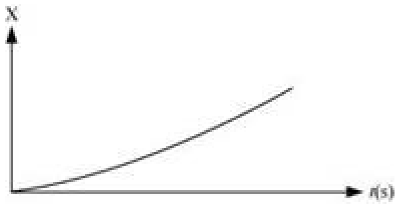
(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its center of mass.

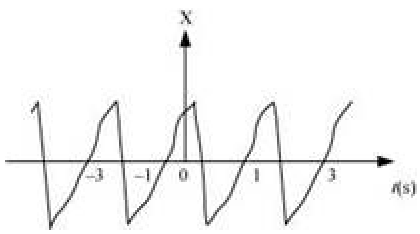
(d) An arrow released from a bow.

7. Figure 14.27 depicts four *x-t* plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

(a)

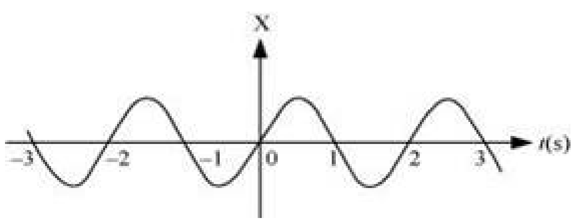


(b)



(c)

(d)



8. Which of the following relationships between the acceleration *a* and the displacement *x* of a particle involve simple harmonic motion?

(a) $a = 0.7x$

(b) $a = -200x^2$

(c) $a = -10x$

(d) $a = 100x^3$

9. The acceleration due to gravity on the surface of moon is 1.7 $\mathrm{ms^{-2}}$. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (*g* on the surface of earth is 9.8 $\mathrm{ms^{-2}}$)

10. A simple pendulum of length *l* and having a bob of mass *M* is suspended in a car. The car is moving on a circular track of radius *R* with a uniform speed *v*. If the pendulum

makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Short Answers-

1. Ans. If T = Time Period of simple pendulum

m = Mass

k = Spring constant

then, $T = 2\pi\sqrt{\frac{m}{k}}$

or $k = \frac{4\pi^2 m}{T^2}$

For first spring : $\rightarrow k_1 = \frac{4\pi^2 m}{t_1^2}$ let $T = t_1$

For second spring : $\rightarrow k_2 = \frac{4\pi^2 m}{t_2^2}$ let $T = t_2$

When springs is connected in parallel, effective spring constant, k = k = $k_1 + k_2$

or $k = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$

If t = total time period

$$\frac{4\pi^2 m}{t^2} = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$$

$$\frac{1}{t^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$$

Or $t^{-2} = t_1^{-2} + t_2^{-2}$

2. Ans. Let y = displacement at any time'‘t’

a = amplitude

w = Angular frequency

v = velocity,

y = a Sin wt

$$v = \frac{dy}{dt} = \frac{d}{dt}(\text{a Sin wt})$$

So, $v = \text{a w Cos wt}$

Now, kinetic energy = K. E. = $\frac{1}{2}mv^2$

$$K.E. = \frac{1}{2}\ m\ w^2a^2\ Cos^2\ wt\ \rightarrow 1)$$

Potential energy = $\frac{1}{2}ky^2$

$$P.E. = \frac{1}{2}ka^2\ Sin^2\ wt\ \rightarrow 2)$$

Adding equation 1) & 2)

Total energy = K.E.+ P.E

$$= \frac{1}{2}mw^2a^2\ Cos^2\ wt + \frac{1}{2}\ ka^2\ Sin^2\ wt$$

$$\text{Since } w = \sqrt{\frac{k}{m}}\ \Rightarrow\ w^2m = k^2$$

$$\text{Total energy} = \frac{1}{2}mw^2a^2Cos^2wt + \frac{1}{2}ka^2Sin^2wt$$

$$= \frac{1}{2}ka^2\ Cos^2\ wt + \frac{1}{2}\ ka^2\ Sin_2\ wt$$

$$= \frac{1}{2}ka^2\ \left(Cos^2\ wt + Sin^2\ wt\right)$$

$$\text{Total energy} = \frac{1}{2}ka$$

Thus total mechanical energy is always constant is equal to $\frac{1}{2}ka^2$. The total energy is independent to time. The potential energy oscillates with time and has a maximum value of $\frac{ka^2}{2}$. Similarly K. E. oscillates with time and has a maximum value of $\frac{ka^2}{2}$. At any instant = constant = $\frac{ka^2}{2}$. The K. E or P.E. oscillates at double the frequency of S.H.M.

3. Ans. Given that a = -b x, Since a $\propto$ x and is directed apposite to x, the particle do moves in S. H. M.

a = b x (in magnitude)

$$or\ \ \frac{x}{a} = \frac{1}{b}$$

$$\text{or}\ \ \frac{Displacement}{Accleration} = \frac{1}{b} \rightarrow 1)$$

$\text{Time period} = T = 2\pi\sqrt{\frac{Displacement}{Accleration}}$

Using equation 1)

$T = 2\pi\sqrt{\frac{1}{b}}$

4. Ans. At t = 0; x = 1cm; w = π|s

t = time

x = Postition

w = Argular frequency

$\therefore$ x = A Cos (Wt + ϕ)

1 = A Cos ($\pi\times0+\phi$)

1 = A Cos ϕ

$Now, v = \frac{dx}{dt} = \frac{d}{dt}(\text{A Cos (wt}+\phi))$

At t = 0 v = π cm|s; w = π|s

$\cancel{\pi}$ = -A$\cancel{\pi}$ Sin ($\pi\times0+\phi$)

$\Rightarrow$ -1 = A Sin ϕ $\rightarrow$ 2)

Squaring and adding 1) & 2)

$A2\ Cos^2\ \phi + A2\ Sin^2\phi = 1+1$

$A2(Cos^2\phi + Sin^2\phi) = 2$

$A^2 = 2$

$A = \sqrt{2}cm$

Dividing 2) by 1), we have :-

$\frac{\cancel{A}\ \text{Sin}\ \phi}{\cancel{A}\ \text{Cos}\ \phi} = -1$

$\tan\phi = -1$

$or\ \ \phi \equiv \tan^{-1}(-1)$

$\phi = \frac{3\pi}{4}$

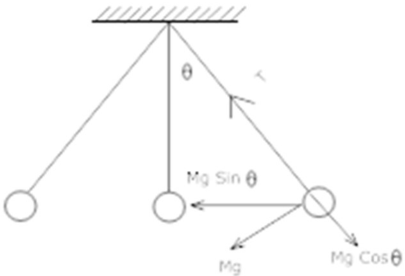
5. Ans. It consist of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support which is free to oscillate.

The distance between point of suspension and point of oscillation is effective length of pendulum.

M = Mass of B ob

x = Displacement = OB

l = length of simple pendulum



Let the bob is displaced through a small angle θ the forces acting on it:-

1) weight = Mg acting vertically downwards.

2) Tension = T acting upwards.

Divide Mg into its components → Mg Cos θ & Mg Sin θ

T = Mg Cos θ

F = Mg Sin θ

- ve sign shows force is divested towards the ocean positions. If θ = Small,

Sin θ $\cong \theta = \frac{Arc\ \text{OB}}{l} = \frac{x}{l}$

$$F = -Mg\frac{x}{l}$$

In S.H.M., vestoring fore, F = -mg θ F = -mg $\frac{x}{l} \rightarrow 1)$

Also, if k = spring constant

F = - k x

$$\cancel{-}\text{mg}\ \frac{\cancel{x}}{l} = \cancel{-}k\ \cancel{x}\left(\text{equating F = - mg}\ \frac{x}{l}\right)$$

$$k = \frac{mg}{l}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{\cancel{m} \times l}{\cancel{m}g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

i.e.1.) Time period depends on length of pendulum and ‘g’ of place where experiment is done.

2) T is independent of amplitude of vibration provided and it is small and also of the mass of bob.

6. Ans. (b) and **(c)**

(a) The swimmer"s motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

7. Ans. (b) and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

8. Ans. (c) A motion represents simple harmonic motion if it is governed by the force law:

$F = -kx$

$ma = -k$

$\therefore a = -\frac{k}{m}x$

Where,

F is the force

m is the mass (a constant for a body)

x is the displacement

a is the acceleration

k is a constant

Among the given equations, only equation *a* = –10 *x* is written in the above form with $\frac{k}{m} = 10$ Hence, this relation represents SHM.

9. Ans. Acceleration due to gravity on the surface of moon, $g' = 1.7\ m\,s^{-2}$

Acceleration due to gravity on the surface of earth, *g* = 9.8 m s^{-2}

Time period of a simple pendulum on earth, *T* = 3.5 s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where,

l is the length of the pendulum

$$\therefore l = \frac{T^2}{(2\pi)^2} \times g$$

$$= \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8\,m$$

The length of the pendulum remains constant.

On moon"s surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi\sqrt{\frac{\frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8}{1.7}} = 8.4\,s$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

10. Ans. The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity = *g*

Centripetal acceleration $= \frac{v^2}{R}$

Where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (a_{eff}) is given as:

$$a_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

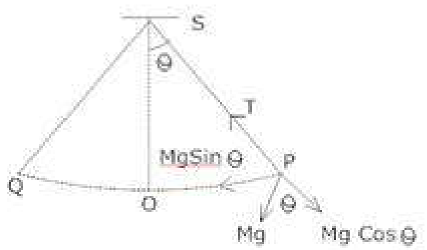
Time period, $T = 2\pi\sqrt{\frac{l}{a_{eff}}}$

Where, *l* is the length of the pendulum

∴Time period, $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$

Long questions-

1.What is Simple pendulum? Find an expression for the time period and frequency of a simple pendulum?



2. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,

(b) at the end B,

(c) at the mid-point of AB going towards A,

(d) at 2 cm away from B going towards A,

(e) at 3 cm away from A going towards B, and

(f) at 4 cm away from B going towards A.

3. The motion of a particle executing simple harmonic motion is described by the displacement function,

x (t) = A cos (ω t + ω).

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s-1. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin (ω t + α), what are the amplitude and initial phase of the particle with the above initial conditions.

4. In Exercise 14.9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

5. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

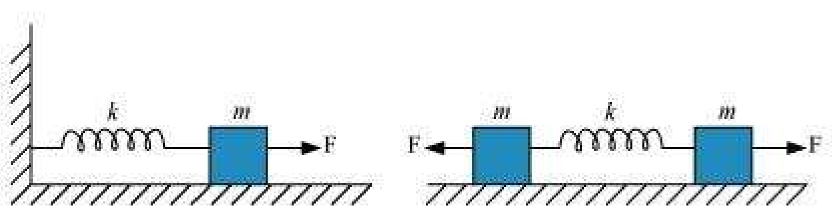
(a) x = -2 sin (3t + π/3)

(b) x = cos (π/6 - t)

(c) x = 3 sin (2πt + π/4)

(d) x = 2 cos πt

6. Figure 14.30 (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.30(b) is stretched by the same force F.

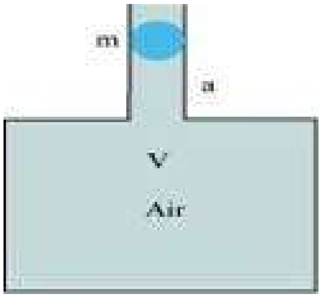


(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

7. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

8. An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.14.33). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal seeFig.14.33.



Long Answers-

1. Ans.A simple pendulum is the most common example of the body executing S.H.M, it consist of heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support, which is free to oscillate.

Let m = mass of bob

l = length of pendulum

Let O is the equilibrium position, OP = X

Let θ = small angle through which the bob is displaced.

The forces acting on the bob are:-

1) The weight = M g acting vertically downwards.

2) The tension = T in string acting along Ps.

Resolving Mg into 2 components as Mg Cos θ and Mg Sin θ,

Now, T = Mg Cos θ

Restoring force F = - Mg Sin θ

- ve sign shows force is directed towards mean position.

Let θ = Small, so Sin θ ≈ θ = $\frac{\text{Arc(op)}}{l} = \frac{x}{l}$

Hence F = - mg θ

F = - mg $\frac{x}{l} \rightarrow 3)$

Now, In S.H.M, F = k x →4) k = Spring constant

Equating equation3) & 4) for F

- k x = - m g $\frac{x}{l}$

Spring factor = k = $\frac{mg}{l}$

Inertia factor = Mass of bob = m

Now, Time period = T

= $2\pi\sqrt{\frac{Inertia\text{ factor}}{Spring\text{ factor}}}$

= $2\pi\sqrt{\frac{m}{mg/l}}$

$T = 2\pi\sqrt{\frac{l}{g}}$

2. Ans.(a) Zero, Positive, Positive

(b) Zero, Negative, Negative

(c) Negative, Zero, Zero

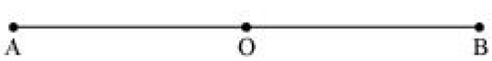
(d) Negative, Negative, Negative

(e) Zero, Positive, Positive

(f) Negative, Negative, Negative

Explanation:

The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.



A particle is in linear simple harmonic motion between the end points

(a) At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is positive as it is directed along AO.

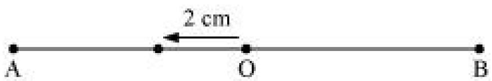
Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point B, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is negative as it is directed along B.

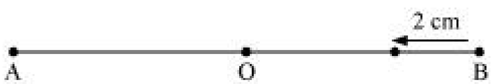
Force is also negative in this case as the particle is directed leftward.

(c)



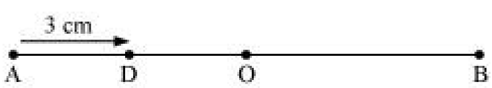
The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d)



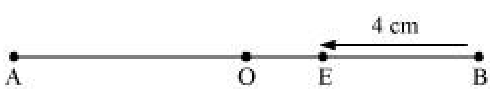
The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A to B. Hence, the particle"s velocity and acceleration, and the force on it are all negative.

(e)



The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Hence, the values for velocity, acceleration, and force are all positive.

(f)



This case is similar to the one given in **(d)**.

3. Ans. Initially, at t = 0:

Displacement, x = 1 cm

Initial velocity, v = ω cm/sec.

Angular frequency, ω = π rad/s^{-1}

It is given that:

$$x(x) = A\cos(\omega t + \phi)$$

$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$

$A\cos\phi = 1$ .....(i)

Velocity, $v = \frac{dx}{dt}$

$\omega = -A\omega \sin(\omega t + \phi)$

$1 = A\sin(\omega \times 0 + \phi) = A\sin\phi$

$A\sin\phi = -1$ ....(ii)

Squaring and adding equations (*i*) and (*ii*), we get:

$A^2(\sin^2\phi + \cos^2\phi) = 1 + 1$

$A^2 = 2$

$\therefore A = \sqrt{2}\, cm$

Dividing equation (*ii*) by equation (*i*), we get:

$\tan\phi = -1$

$\therefore \phi = \frac{3\pi}{4}, \frac{7\pi}{4}, ...$

SHM is given as:

$x = B\sin(\omega t + a)$

Putting the given values in this equation, we get:

$1 = B\sin[\omega \times 0 + a]$

$B\sin a = 1$ ....(iii)

Velocity, $v = \omega B\cos(\omega t + a)$

Substituting the given values, we get:

$\pi = \pi B\sin a$

$B\sin a = 1$ ....(iv)

Squaring and adding equations (*iii*) and (*iv*), we get:

$B^2[\sin^2 a + \cos^2 a] = 1 + 1$

$B^2 = 2$

$\therefore B = \sqrt{2}\, cm$

Dividing equation (*iii*) by equation (*iv*), we get:

$$\frac{B\sin a}{B\cos a}=\frac{1}{1}$$

$$\tan a=1=\tan\frac{\pi}{4}$$

$$a\frac{\pi}{4},\frac{5\pi}{4},\dots\dots\dots$$

4. Ans. (a) $x = 2\sin 20t$

(b) $x = 2\cos 20t$

(c) $x = -2\cos 20t$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, $A = 2.0$ cm

Force constant of the spring, $k = 1200$ N $\mathbf{m}^{-1}$

Mass, $m = 3$ kg

Angular frequency of oscillation:

$$\omega=\sqrt{\frac{k}{m}}$$

$$=\sqrt{\frac{1200}{3}}=\sqrt{400}=20\,rad\,s^{-1}$$

(a) When the mass is at the mean position, initial phase is 0.

Displacement, $x = A\sin \omega t$

$= 2\sin 20t$

(b) At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

Displacement, $x=A\sin\left(\omega t+\frac{\pi}{2}\right)$

$$=2\sin\left(20t+\frac{\pi}{2}\right)$$

$= 2\cos 20t$

(c) At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

Displacement, $x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$

$= 2\sin\left(20t + \frac{3\pi}{2}\right)$

$= -2\cos 20t$

The functions have the same frequency $\left(\frac{20}{2\pi}Hz\right)$ and amplitude (2 cm), but different initial phases $\left(0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$.

5. Ans.(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right) = +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

$= 2\cos\left(3t + \frac{5\pi}{6}\right)$

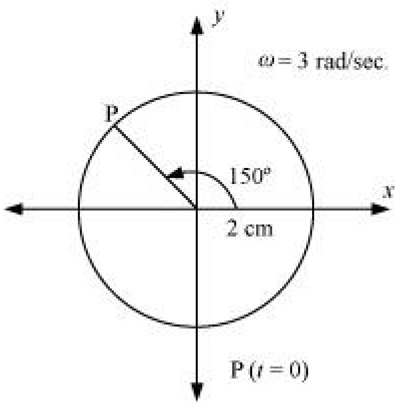
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi = \frac{5\pi}{6} = 150°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3\,rad/\sec$.

The motion of the particle can be plotted as shown in the following figure.



(b) $x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$

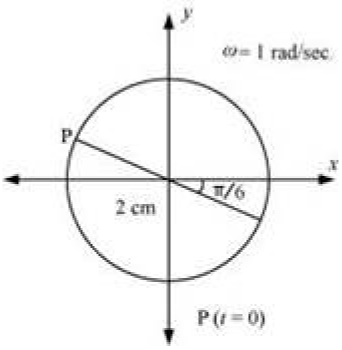
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A=2

Phase angle, $\phi = \frac{\pi}{6} = 30°$

Angular velocity, $\omega = \frac{2\pi}{T} = 1 rad/\sec.$

The motion of the particle can be plotted as shown in the following figure.



(c) $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$$

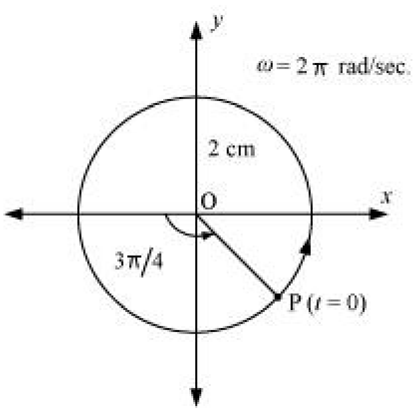
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 3 cm

Phase angle, $\phi = \frac{3\pi}{4}$ = 135°

Angular velocity, $\omega = \frac{2\pi}{T} = 2\pi rad/\sec.$

The motion of the particle can be plotted as shown in the following figure.



(d) $x = 2\cos \pi t$

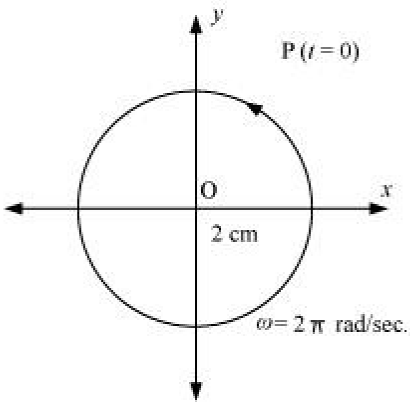
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, ϕ = 0

Angular velocity, ω = π rad/s

The motion of the particle can be plotted as shown in the following figure.



6. Ans.(a) For the one block system:

When a force F, is applied to the free end of the spring, an extension l, is produced. For the maximum extension, it can be written as:

$F = kl$

Where, k is the spring constant

Hence, the maximum extension produced in the spring, $l = \frac{F}{k}$

For the two block system:

The displacement (x) produced in this case is:

$$x = \frac{1}{2}$$

Net force, $F = +2\,kx$ $2k\frac{1}{2}$

$$\therefore l = \frac{F}{k}$$

(b) For the one block system:

For mass (m) of the block, force is written as:

$$F = ma = m\frac{d^2x}{dt^2}$$

Where, x is the displacement of the block in time t

$$\therefore m\frac{d^2x}{dt^2} = -kx$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x = -\omega^2 x$$

Where, $\omega^2 = \frac{k}{m}$

$$\omega = \sqrt{\frac{k}{m}}$$

Where,

ω is angular frequency of the oscillation

∴Time period of the oscillation, $T = \frac{2\pi}{\omega}$

$$= \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

For the two block system:

$$F = m\frac{d^2x}{dr^2}$$

$$m\frac{d^2x}{dr^2} = -2kr$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dr^2} = -\left[\frac{2k}{m}\right]x = -\omega^2 x$$

Where,

Angular frequency, $\omega = \sqrt{\frac{2k}{m}}$

∴Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

7. Ans. Area of cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

$F = -(\text{Volume} \times \text{Density} \times g)$

$F = -(A \times 2h \times \rho \times g) = -2\rho gh = -k \times$ Displacement in one of the arms (h)

Where,

$2h$ is the height of the mercury column in the two arms

k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where,

m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube.

Mass of mercury, m = Volume of mercury $\times$ Density of mercury

$= Al\rho$

$\therefore T = 2\pi\sqrt{\frac{m}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$

Hence, the mercury column executes simple harmonic motion with time period $2\pi\sqrt{\frac{l}{2g}}$.

8. Ans. Volume of the air chamber = V

Area of cross-section of the neck = a

Mass of the ball = m

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\Delta V = ax$

Volumetric strain $= \frac{\text{Change in volume}}{\text{Original valume}}$

$\Rightarrow \frac{\Delta V}{V} = \frac{ax}{V}$

Bulk Modulus of air, $B = \frac{Stress}{Strain} = \frac{-p}{\frac{ax}{V}}$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$$p = \frac{-Bax}{V}$$

The restoring force acting on the ball, $F = p \times a$

$$\frac{-Bax}{V} \cdot a$$

$$= \frac{-Ba^2x}{V}$$

In simple harmonic motion, the equation for restoring force is:

$F = -kx$ … (ii)

Where, k is the spring constant

Comparing equations (i) and (ii), we get:

$$= \frac{Ba^2}{V}$$

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

Assertion Reason Answer:

1. (a) If both assertion and reason are true but reason is not the correct explanation of the assertion.
2. (d) If the assertion and reason both are false.

Case Study Questions-

1. A motion that repeats itself at regular intervals of time is called periodic motion. Very often, the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from the position, a force comes into play which tries to bring the body periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory. The smallest interval of time after which the motion is repeated is called its period. Let us denote the period by the symbol T. Its SI unit is second. The reciprocal of T gives the number of repetitions that occur per unit time. This

quantity is called the frequency of the periodic motion. It is represented by the symbol n. The waves, Heinrich Rudolph Hertz (1857–1894), a special name has been given to the unit of frequency. It is called hertz (abbreviated as Hz). Answer the following.a)

i. Every oscillatory motion is periodic motion true or false?
   a. True
   b. False
ii. Circular motion is
   a. Oscillatory motion
   b. Periodic motion
   c. Rotational motion
   d. None of these
iii. Define period. Give its SI unit and dimensions
iv. Define frequency of periodic motion. How it is related to time period
v. What is oscillatory motion

2. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations. All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called forced or driven oscillations. We consider the case when the external force is itself fact of forced periodic oscillations is that the system oscillates not with its natural frequency ω, but at familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations. The maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping, and is never infinity. The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.

i. When a system oscillates with its natural frequency ω, and the oscillations are called
   a. Free oscillations
   b. Forced oscillations
ii. All free oscillations eventually die out because of
   a. Damping force
   b. electromagnetic force
   c. None of these

iii. What is free oscillation?

iv. What is forced oscillations?

v. What is resonance?

Case Study Answer-

1. Answer

i. (a) True

ii. (b) Periodic motion

iii. The smallest interval of time after which the motion is repeated is called its period. Its SI unit is second and dimensions are [T1].

iv. Reciprocal of Time period (T) gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The relation between n and T is n = 1/T i.e. they are inversely proportional to each other. The unit of n is thus s-1 or hertz.

v. Oscillatory motion is type of periodic motion in which body performs periodic to and fro motion about some mean position. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.

2. Answer

i. (a) Free oscillations

ii. (b) Damping force

iii. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations.

iv. Forced oscillations are oscillations where external force drives the oscillations with frequency given by external force.

v. The phenomenon of increase in amplitude when the driving force is close to encounter phenomena which involve resonance. Your experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.